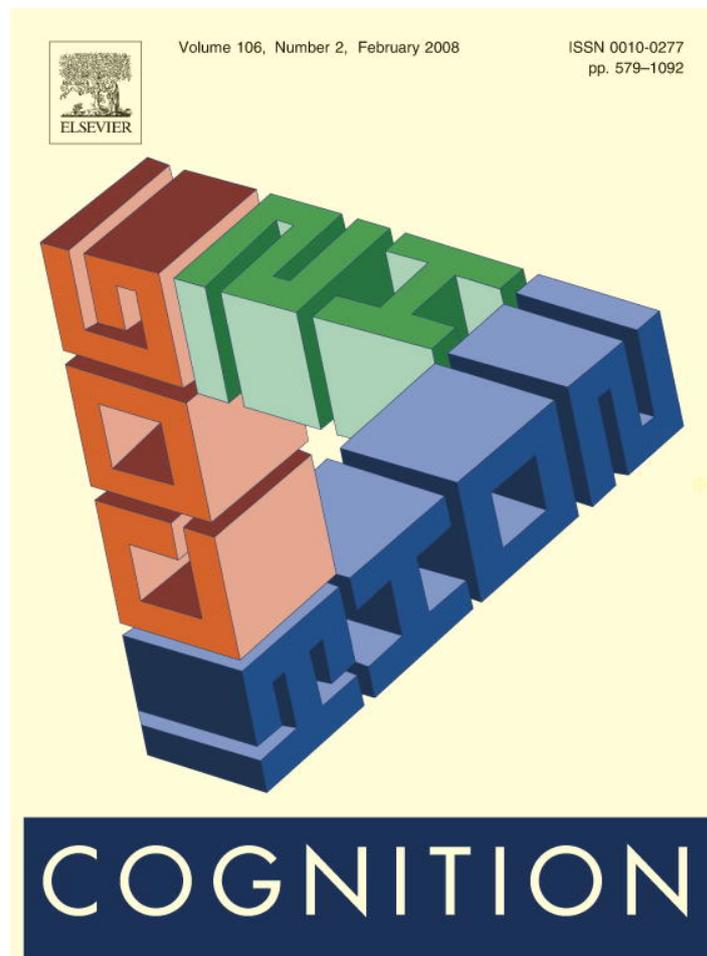


Provided for non-commercial research and education use.  
Not for reproduction, distribution or commercial use.



This article was published in an Elsevier journal. The attached copy is furnished to the author for non-commercial research and education use, including for instruction at the author's institution, sharing with colleagues and providing to institution administration.

Other uses, including reproduction and distribution, or selling or licensing copies, or posting to personal, institutional or third party websites are prohibited.

In most cases authors are permitted to post their version of the article (e.g. in Word or Tex form) to their personal website or institutional repository. Authors requiring further information regarding Elsevier's archiving and manuscript policies are encouraged to visit:

<http://www.elsevier.com/copyright>

Available online at [www.sciencedirect.com](http://www.sciencedirect.com) ScienceDirect

COGNITION

Cognition 106 (2008) 994–1003

[www.elsevier.com/locate/COGNIT](http://www.elsevier.com/locate/COGNIT)

Brief article

# Numeric comparison in a visually-guided manual reaching task <sup>☆</sup>

Joo-Hyun Song <sup>\*</sup>, Ken Nakayama*Department of Psychology, Harvard University, USA*

Received 14 May 2006; revised 21 March 2007; accepted 25 March 2007

---

## Abstract

Nearly all studies on perception and cognition have used discrete responses to infer internal cognitive processes. In the current study, we demonstrate that visually-guided manual reaching can provide new opportunities to access internal processes over time. In each trial, participants were required to compare a single digit Arabic number presented on the center square with the standard, 5. Participants were asked to reach and touch one of three squares on the screen with their index finger while their hand movement trajectories were recorded: the left square for 1–4, the center for 5, and the right for 6–9. Direct evidence for an analogue representation of numbers was found in early as well as in later portions of hand trajectories, showing systematic shifts in position for small differences in numerical magnitude.

© 2007 Elsevier B.V. All rights reserved.

*Keywords:* Visually-guided manual pointing; Numeric distance effect; Number line; Curved trajectories; Initial movement direction

---

## 1. Introduction

For decades, various aspects of human cognitive and perceptual processes have been inferred based on reaction times and accuracies obtained from discrete

---

<sup>☆</sup> This manuscript was accepted under the editorship of Jacques Mehler

<sup>\*</sup> Corresponding author. Address: The Smith-Kettlewell Eye Research Institute, 2318 Fillmore Street, San Francisco, CA 94115, USA. Tel.: +1 415 345 2061; fax: +1 415 345 8455.

E-mail address: [jhsong@ski.org](mailto:jhsong@ski.org) (J.-H. Song).

responses such as key pressing. Yet, these discrete responses cannot represent internal processes developing over time. One of our aims is to support the view that human cognitive and perceptual processes are continuous and dynamic even when the responses required of the participants are discrete and categorical. Although this dynamic view is generally accepted in neurophysiology where continuous temporal dynamics of neural activities can be directly observed, these are not usually available in studies on human perception and cognition (see Spivey & Dale, 2004 for review).

However, recent studies have begun to shed light on the importance of dynamic aspects in cognitive processes. For instance, Spivey, Grosjean, and Knoblich (2005) suggested that continuous measurements of hand movements more directly reveal internal language processes. They asked participants to drag a mouse to an auditorily specified target picture, while a distractor picture was presented on the opposite side. Mouse-tracking trajectories were attracted more toward a distractor when the target and distractor were from the same lexical cohort such as “*candle*” and “*candy*”. This demonstrated a dynamic online competition between simultaneously activated lexical representations (Magnuson, 2005). Boulenger et al. (2006) also examined an interaction between language processes and overt motor behavior. They showed that depending on whether action words were processed before or after the movement onset, reaching movements were either interfered or facilitated.

Song and Nakayama (2006) demonstrated a similar advantage using a visually-guided reaching task. Goal-directed reaching trajectories obtained from participants pointing to an odd-colored target among distractors reflected ongoing changes in focal attention allocation. Highly curved trajectories were directed first towards a distractor and quickly corrected to the target, showing that motor plans compete even after these rapid movements are initiated. Therefore, these studies further support that action tasks can reveal internal states as they unfold over time.

In addition, results from goal-directed action tasks are interesting because neuropsychological patients (Milner & Goodale, 1992, 1995; Stoerig & Cowey, 1997), and normal participants (Aglioti, DeSouza, & Goodale, 1995; Haffenden & Goodale, 1998) have shown functional and anatomical dissociation between visual perception and spatial-motor action. For instance, a patient with apperceptive agnosia (Milner & Goodale, 1992, 1995) can still perform appropriate motor actions such as reaching and grasping for objects, even when they cannot see them. Thus, action tasks provide hidden aspects that are not revealed in perception and cognitive tasks alone.

In the current study, we introduced a goal-directed reaching task, and generalized the applicability of continuous measurements to the mental representation of numbers. In each trial, three squares were presented on a screen, and the center square contained an Arabic digit between 1 and 9. Participants were asked to execute manual pointing as quickly as possible to the left square for numbers smaller than 5, the right for numbers larger than 5, or to the center for 5 while their hand movement trajectories were recorded (Fig. 1).

Moyer and Landauer (1967) were the first to report that reaction times and error rates systematically decrease as the numerical distance between two numbers increases. This *numeric distance effect* has been observed in adults as well as infants and animals (Antell & Keating, 1983; Dehaene, Dupoux, & Mehler, 1990; Gallistel &

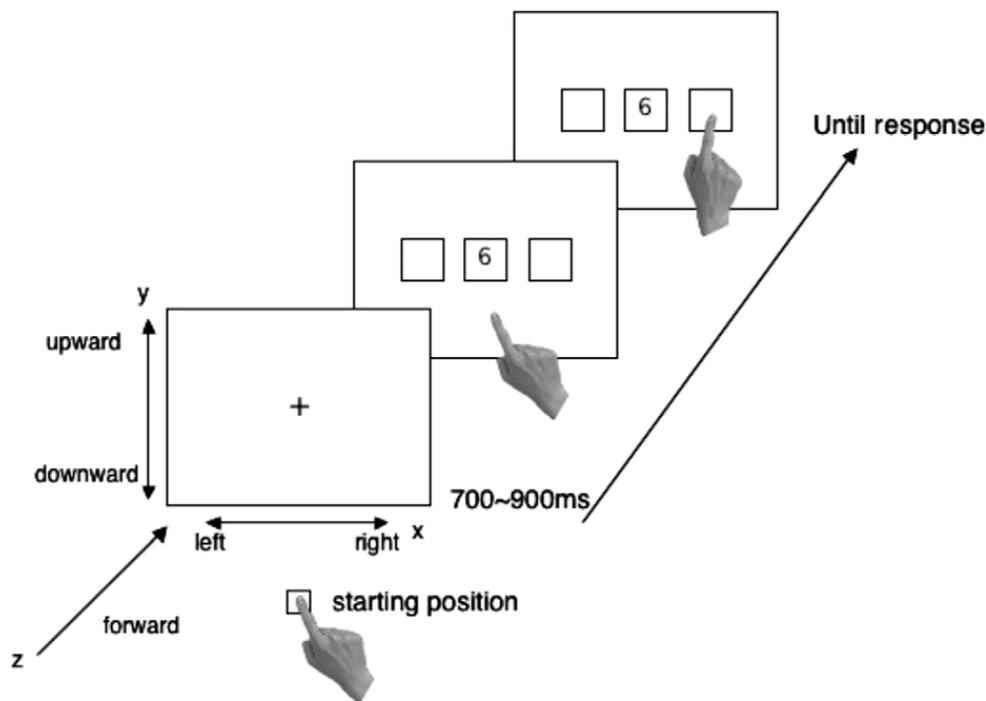


Fig. 1. Number comparison task. Participants were asked to touch the left square for numbers smaller than 5, the right square for those larger than 5, or the center for 5.  $x$  (left–right),  $y$  (upward–downward) and  $z$  (forward) axes indicate three directions of reaching movements.

Gelman, 1992; Xu & Spelke, 2000). The distance effect that can be seen with arbitrary symbols such as Arabic digits supports the idea that numbers are automatically encoded based on their analogical quantity, and that the proximity relations between them are spatially preserved along the mental number line (Dehaene, Bossini, & Giraux, 1993; Dehaene et al., 1990; Moyer & Landauer, 1967). This effect is also seen even when the numeric comparison was not obligatory such as in same-different matching or priming tasks (Dehaene & Akhavein, 1995). Furthermore, numerous studies have claimed that the mental number line has an orientation such that small numbers are represented to the left and large ones are to the right (e.g., Dehaene et al., 1993, 1990).

The characteristics of number representations have been typically examined with discrete responses such as key pressing. Patterns of reaction times in numeric comparison tasks have critically contributed to resolving a variety of issues about how numbers are represented in the brain. However, this is an indirect inference based on the relative duration for completing an entire sequence of processes, and does not reveal the intermediate processes occurring over time. Some studies have attempted to investigate spatial representations of numbers more directly.

For instance, Fischer and colleagues applied goal-directed saccades or pointing to number parity tasks. In accordance with previous studies, they also showed that saccades and manual pointing were executed faster when the left target was assigned to

small digits and the right target to large digit. These studies, therefore, further demonstrate the association of number magnitudes with spatial response codes (Fischer, 2003; Fischer, Warlop, Hill, & Fias, 2004). In addition, Ishihara et al. (2006) asked participants to reach for a target presented in one of five horizontally varying locations when the target was an odd number. They found that reaction time decreased as the congruity between target locations and number values increased, claiming that there is a continuous mapping between space and number representations. Yet, because they did not find a relationship between number representations and movement time, they concluded that this interference of number processing on action is limited to motor preparation.

In all of these studies, however, movement trajectories were not examined. Our endeavor in the current study is to map heretofore invisible internal cognitive processes of numeric comparison by measuring such trajectories in three dimensions.

## 2. Methods

### 2.1. Participants

Nine right-handed Harvard University students participated in the study for course credit. They all had normal or corrected-to-normal visual acuity. All experimental procedures were approved by the Harvard University Institutional Review Board.

### 2.2. Stimuli

The visual display was back projected on an upright plexi-glass screen (43 cm × 35 cm). Three white unfilled squares ( $2^\circ \times 2^\circ$ ) were equally spaced horizontally against a black background. The middle square was located on the center of the screen and the centers of the other two squares were  $12^\circ$  left or right from the center of the middle square. A white single Arabic digit (1–9) was presented on the center square (Fig. 1).

### 2.3. Task

Participants were tested individually in a semi-darkened room. They were seated 48 cm in front of the visual display. In each trial, a black screen with a white cross fixation mark was presented for 700–900 ms. Then three squares were presented, with the center square containing an Arabic digit between 1 and 9. Participants were asked to execute manual pointing as quickly as possible to the left square for numbers smaller than 5, to the right for those larger than 5, or to the center for 5 (standard), while their hand movement trajectories were recorded (Fig. 1). Because there could be interference with left–right reading and typical number line arrangement, we did not counter balance the left–right response rule (Dehaene et al., 1993). When participants touched one of the squares, a beep indicated whether participants

selected the correct response and the display disappeared. The inter-trial interval was 1000ms. Participants performed 5 blocks, each consisting of 64 trials. In total, the numeric 5 appeared as the target in 160 trials, and the other eight numbers were presented 20 times each as the target. Thus, participants were exposed to the equal numbers of standard and non-standard targets. Participants practiced 60 trials before the experiment, consisting of 2 demonstration trials each for the three groups, and 54 trials presenting the numeric 1–9 equally as the target.

#### 2.4. *Measuring hand movements*

Hand movements were tracked with a Fastrak electromagnetic position and orientation measuring system (Polhemus Inc.) with a sampling rate of 120 Hz. The small position-tracking sensor (0.89" × 0.50" × 0.45") was attached to the tip of the right hand index finger. The starting position (3 cm × 3 cm) was marked on the table, which was approximately aligned with the body midline and 27 cm in front of the participants. Participants were required to put their index finger on the starting position to initiate each trial. The tracking system was calibrated for each block with nine distributed points.

#### 2.5. *Data analysis*

Movement data were transmitted to a Power Mac G4 by Vision Shell library ([www.visionshell.com](http://www.visionshell.com)) for off-line analysis to identify the onset and offset of movements. Hand velocity exceeding a threshold of 10 cm/s demarcated the onset of the movement. Each trajectory was visually inspected to verify the appropriateness of this criterion, and was adjusted by hand if necessary. Manual adjustment was needed very rarely, in less than 1–2 trials per participant. Only trials in which participants touched the correct target were included for further analysis.

*Total time* was defined as the interval between stimulus onset and the end of pointing movements, divided into two components: reaction time and movement time. *Reaction time* was defined as the interval between stimulus onset and the beginning of the movement and *movement time* was defined as the interval between movement onset and the end of pointing movement. Trials in which movements were initiated earlier than 100 ms or completed in excess of 1500 ms after the target onset were excluded as anticipatory movements or outliers. Less than 2% of the trials were eliminated because of this criterion. *Initial movement direction* was the angular deviation from the mean trajectory toward the center square.

### 3. Results

First, we will briefly describe results from the analysis of conventional time and accuracy indexes and then focus on the movement trajectories. As shown in [Table 1](#), total times, the interval between stimulus onset and the end of pointing movements, decreased as the target–standard distance increased and this comparison

Table 1

Mean reaction time, movement time, total time, and accuracy as a function of target–standard distances (*SE*)

	Reaction time (ms)	Movement time (ms)	Total time (ms)	Accuracy (%)
Distance 1	338(42)	497(28)	836(55)	92(3)
Distance 2	327(49)	495(26)	822(49)	93(3)
Distance 3	328(39)	489(29)	817(51)	93(4)
Distance 4	320(38)	488(29)	808(51)	96(3)
Standard	313(33)	474(28)	787(48)	93(3)

almost reached statistical significance,  $F(3, 24) = 2.98$ ,  $p = .051$ . Total times were divided into *reaction times* (between stimulus onset and the beginning of the movements) and *movement times* (between movement onset and the end of pointing movements). Reaction times and movement times showed the same decreasing linear trend as the target–standard distance increased as in total times but it was not statistically significant,  $ps > .1$ . There was no difference in accuracies for the numeric distance conditions,  $F(3, 24) = 1.87$ ,  $p < .2$ . Consistent with previous studies (see Dehaene, 1997 for review), we observed the trend of the numeric distance effect in these measurements although these results were much less obvious than trajectory measurements, which we describe below.

Movement trajectories of participants performing the numeric comparison task demonstrated numeric distance effects. Fig. 2 shows trajectories of typical responses to the numbers 1–5. When the target was 5 (standard), movements were directed towards the center square (Fig. 2a). Fig. 2b–e demonstrate that there was a systematic shift of initial trajectories towards a hypothetical position on a number line intermediate between the numeral 1 and 5 positions (straight ahead). For example, when a numeral 1 was presented, the trajectory was straight as the finger moved to the correct square (left). However, when a numeral 4 was presented, there are examples where a movement was directed closer to the center of the display, swerving along a curved path to the correct response square. Thus, it showed that as the numeric distance decreased, movements were curved more towards the center response square. This progressive deviation provides additional and more direct evidence for the existence of a mental number line. From the traces, it is evident that this deviation can occur early in the trajectory.

In order to systematically investigate this number line representation, we examined the evolution of trajectories over time. For further analyses, all responses were collapsed based on the target–standard distance, irrespective of direction. For example, left and right responses to the number 1 and the number 9 were collapsed as the target–standard distance 4.

For each participant, we averaged movement trajectories based on target–standard numeric distances. Following Spivey et al. (2005), trajectories were individually normalized by resampling 101 equally spaced time points (0–100%) during movement time. Corresponding  $x$ ,  $y$ ,  $z$  positions were computed by the linear interpolation. Fig. 3 depicts trajectory deviations in the  $x$  direction (left–right) from the standard for each of the four target–standard distances. Higher values along the

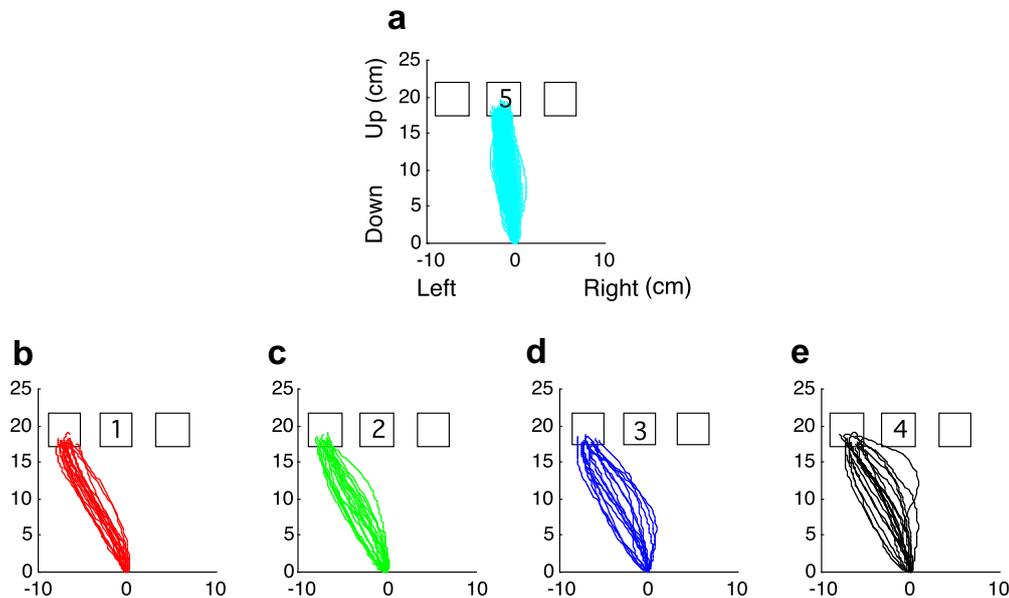


Fig. 2. Manual reaching trajectories for the standard ((a), center square), and targets smaller than the standard ((b–e), left square). This is an example from a typical participant. The participant usually made direct movements to the center square (a). In contrast, for numbers between 1 (b) and 4 (e), movement trajectories are on average less direct to the left square and more curved towards the center square as the target–standard numeric distance decrease. These trajectories are three-dimensional, but for clarity we only show the  $x$  (left–right) and  $y$  (upward–downward) dimensions, which are most relevant for the numeric distance effect. Response squares are not drawn to scale for demonstration purpose.

$y$ -axis indicate trajectories that are more deviated from trajectories toward the center square. Fig. 3 shows that as the target–standard distance increases, the mean trajectory is deviated more from the standard trajectory.

To document how these differences unfold over time, in every 5% time slice (5–100%), we conducted repeated ANOVA on the deviation from the standard trajectory in the four target–standard distance conditions. We found statistically significant numeric distance effects in the  $x$  direction ( $p < .05$ ) continuously from 30% to 95%. The endpoint of reaching (100%) were also linearly separated according to the numeric line,  $F(3, 24) = 3.78$ ,  $p < .025$  (Fig. 3 inset), consistent with Fischer (2001), in which number processing led to spatial biases when participants were asked to bisect numeric strings.

During this period (30–100%), the linear contrasts were also all significant ( $p < .005$ ). Thus, the first indication of the numeric line (30%) was  $\approx 480$  ms after the target onset, that is,  $\approx 150$  ms after the movement initiation. Furthermore, *Initial movement direction*, which was the angular deviation from the direction of standard trials, also confirmed that at 150 ms after the movement onset, trajectory directions were deviated linearly more from the standard as the numeric distance increased,  $F(3, 24) = 13.0$ ,  $p < .001$ . Mean initial movement direction for target–standard distances 1–4 were 12 ( $SE = 1.6$ ), 14(1.7), 15(1.6), and 16(1.4) degrees, respectively.

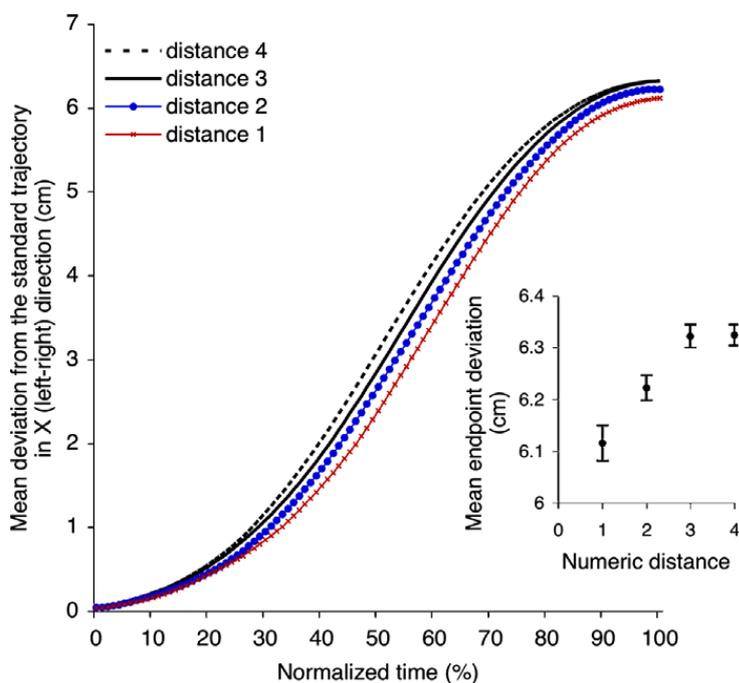


Fig. 3. Mean trajectory deviations from the standard trajectory across all participants. The inset shows mean deviations in endpoints from that of the standard condition as a function of the target–standard numeric distance.

Therefore, these results demonstrated that movement trajectories were deviated more from the standard as the numeric distance increased. This finding is the shortest latency numeric magnitude effect reported. Previous studies using key pressing reaction times (Moyer & Landauer, 1967; Dehaene et al., 1993, 1990) typically showed much longer latencies because they measure the endpoint of the process, including the final motor output. We think that this early detection is possible because our task allowed in-flight correction.

Taken together, these results demonstrate that with this relatively small number of participants, continuous trajectories afford more sensitive and robust results than previous methods, also providing new information about internal processes as they evolve over time.

#### 4. Discussion

In the current study, characteristics of trajectories showed that numeric magnitude of the target is spatially encoded, and the proximity and orders between numbers are spatially represented along a hypothesized mental number line. The greater the numeric deviation, the greater is the deviation of the trajectory. It is worth noting that there are no visual stimuli in the left and right boxes. Thus, the differences in the

initial direction and the curved trajectories reflect internal competition processes between numeric representations rather than perceptual competition.

Compared to typical perceptual or cognitive tasks requiring discrete responses such as button pressing or verbal reporting, our results provided more direct evidence for a spatial number representation. Instead of inferring intermediate cognitive processes based on the final reaction times, action tasks can provide information about internal states as they unfold over time. We also found that the influence of the continuous numeric line is not limited to the motor preparation stage, but also altered entire movement trajectories. In addition, with a small number of participants, we empirically observed that trajectory measurements could be more robust and sensitive than discrete time measurements for revealing ongoing cognitive processes.

Taken together, the close coupling of higher level cognitive processes and the details of overt behavior suggest new applications. Details of trajectories could be efficacious for various populations such as young children, clinical patients or animals. For example, in our previous visually-guided reaching studies, we found that pointing trajectories reveal the site of focal attention as it moves from one item to another (Song & Nakayama, 2006) as well as revealing a competitive motor program elicited by a masked word, which participants were not aware of (Finkbeiner, Song, Nakayama, & Caramazza, in press).

### Acknowledgements

J.H.S. is supported by a Korea Foundation for Advanced Studies fellowship. We thank Rachel Keen, Irene Pepperberg, Michael Spivey, Susan Carey, Matthieu LeCorre, and Martin Fischer for helpful comments.

### References

- Aglioti, S., DeSouza, J. F. X., & Goodale, M. A. (1995). Size-contrast illusions deceive the eye but not the hand. *Current Biology*, *5*, 679–685.
- Antell, S., & Keating, D. (1983). Perception of numerical invariance in neonates. *Child Development*, *54*, 695–701.
- Boulenger, V., Roy, A. C., Paulignan, Y., Deprez, V., Jeannerod, M., & Nazir, T. A. (2006). Cross-talk between language processes and overt motor behavior in the first 200 ms of processing. *Journal of Cognitive Neuroscience*, *18*(10), 1607–1615.
- Dehaene, S. (1997). *The number sense*. New York: Oxford University Press.
- Dehaene, S., & Akhavan, R. (1995). Attention, automaticity and levels of representation in number processing. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *21*, 314–326.
- Dehaene, S., Bossini, S., & Giraux, P. (1993). The mental representation of parity and numerical magnitude. *Journal of Experimental Psychology: General*, *122*, 371–396.
- Dehaene, S., Dupoux, E., & Mehler, J. (1990). Is numerical comparison digital? Analogical and symbolic effects in two-digit number comparison. *Journal of Experimental Psychology: Human Perception and Performance*, *16*, 626–641.
- Finkbeiner, M., Song, J. -H., Nakayama, K., & Caramazza, A. (in press). Engaging the motor system with masked orthographic primes: A kinematic analysis. *Visual Cognition*.
- Fischer, M. H. (2001). Number processing induces spatial performance biases. *Neurology*, *57*(5), 822–826.

- Fischer, M. H. (2003). Spatial representation in number processing – Evidence from a pointing task. *Visual Cognition*, 10(4), 493–508.
- Fischer, M. H., Warlop, N., Hill, R. L., & Fias, W. (2004). Oculomotor bias induced by number perception. *Experimental Psychology*, 51(2), 91–97.
- Gallistel, C. R., & Gelman, R. (1992). Preverbal and verbal counting and computation. *Cognition*, 44, 43–74.
- Haffenden, A. M., & Goodale, M. A. (1998). The effect of pictorial illusion on prehension and perception. *Journal of Cognitive Neuroscience*, 10, 122–136.
- Ishihara, M., Jacquin-Curtois, S., Flory, V., Salemme, R., Imanaka, K., & Rossetti, Y. (2006). Interaction between space and number representations during motor preparation in manual aiming. *Neuropsychologia*, 44(7), 1009–1016.
- Magnuson, J. S. (2005). Moving hand reveals dynamics of thought. *Proceedings of the National Academy of Sciences*, 102, 9995–9996.
- Milner, A. D., & Goodale, M. A. (1992). Separate visual pathways for perception and action. *Trends in Neuroscience*, 15, 20–25.
- Milner, A. D., & Goodale, M. A. (1995). *The visual brain in action*. Oxford: Oxford University Press.
- Moyer, R. S., & Landauer, T. K. (1967). Times required for judgments of numerical inequality. *Nature*, 215, 1519–1520.
- Song, J.-H., & Nakayama, K. (2006). Role of focal attention on latencies and trajectories of visually-guided manual pointing. *Journal of Vision*, 6(9):11, 982–995. doi:10.1167/6.9.11. Available from <http://journalofvision.org/6/9/11/>.
- Spivey, M., & Dale, R. (2004). On the continuity of mind: Toward a dynamical account of cognition. In B. Ross (Ed.). *The Psychology of Learning and Motivation* (Vol. 45). Elsevier.
- Spivey, M., Grosjean, M., & Knoblich, G. (2005). Continuous attraction toward phonological competitors: Thinking with your hands. *Proceedings of the National Academy of Sciences*, 102, 10393–10398.
- Stoerig, P., & Cowey, A. (1997). Blindsight in man and monkey. *Brain*, 120, 535–559.
- Xu, F., & Spelke, E. S. (2000). Large number discrimination in 6-month-old infants. *Cognition*, 74, B1–B11.